Mirrors in Computer Graphics, Computer Vision and Time-of-Flight Imaging

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Abstract. Mirroring is one of the fundamental light/surface interactions occurring in the real world. Surfaces often cause specular reflection, making it necessary to design robust geometry recovery algorithms for many practical situations. In these applications the specular nature of the surface is a challenge. On the other side, mirrors, with their unique reflective properties, can be used to improve our sensing modalities, enabling applications such as surround, stereo and light field imaging. In these scenarios the specular interactions are highly desirable. Both of these aspects, the utilization and circumvention of mirrors are present in a significant amount of publications in different scientific areas. These publications are covering a large number of different problem statements as well as many different approaches to solutions. In the chapter we will focus on a collection and classification of the work in this area.

1 Introduction

Apart from refraction and diffraction, mirroring is one of the fundamental means for shaping light distributions, either for imaging or for projection purposes. Whereas refractive, or *dioptric*, systems, mainly in the form of camera optics, are widely employed in the computer vision literature, *cataoptric*, or mirror systems have mainly been used in the design of large scale optics where refractive elements are impractical, e.g. for telescopes. The combination of refractive and mirror elements in imaging and measurement systems is known as *catadioptric* imaging.

In this chapter, we review the design and application of mirror systems in computer graphics and computer vision, as well as the related problem of the determination of the geometry of a mirror or mirror system. While less obvious, we point out a connection between mirror calibration or mirror shape estimation and time-of-flight imaging.

Our methodology is based on a classification scheme for mirror systems, Fig. 1, that builds on the fundamental imaging properties of the employed mirror surfaces. We categorize existing systems into classes based on their mirroring properties and their use in active or passive imaging systems. The main categories for mirror systems are whether the mirrors are planar or curved, whether single or multiple mirrors are used and whether single-bounce or multi-bounce interaction is employed.

We first discuss the different classes with respect to their imaging properties, Sect. 2, and introduce the tool of ray unfolding for doing so. Next, we discuss passive imaging devices that utilize mirrors, Sect. 3. Passive systems have the property that light rays that cover a common scene point do not influence each other. On the other hand, if active illumination is introduced, light can super-position in a scene. We discuss

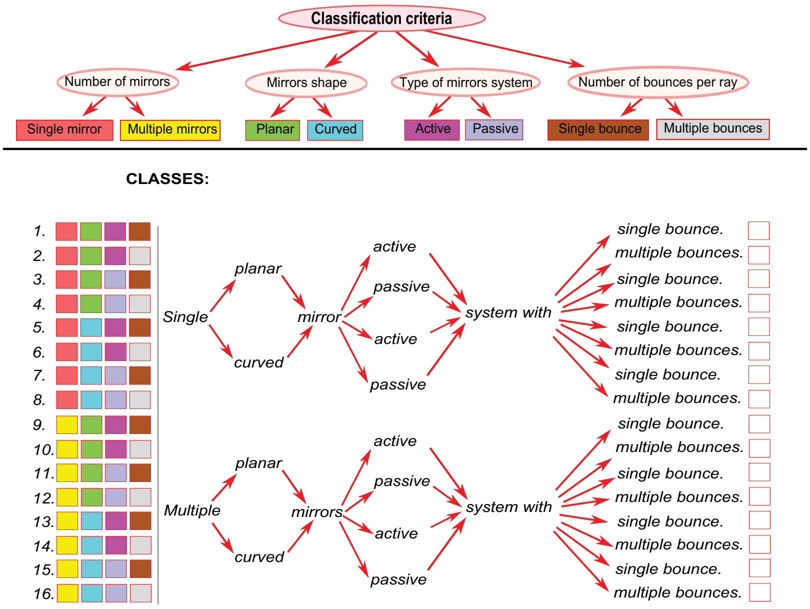

Fig. 1: Classification

active imaging systems in Sect. 4. All systems involving mirrors need to be calibrated, i.e. the geometry and position of the mirrors in the scene has to be determined. For this reason, we review computer vision methods that aim at determining the shape of specular reflective surfaces or the position of a camera with respect to a known mirror geometry in Sect. 5. The recovery of a mirror system's geometry from depth measurements is a special case of the calibration problem. However, this problem has its own literature and approaches in the field of time-of-flight imaging and acoustics. We therefore draw connections between the previously discussed techniques and the time-of-flight literature in Sect. 6. Finally, we summarize the article and formulate important open questions, that in our opinion, must be solved in order to achieve further progress in the area of mirror systems.

2 Classification Scheme and Mirror System Interpretation

Here we present our classification scheme, Fig. 1, in conjunction with a discussion of the main properties of the mirror systems involved. The two main classes are planar mirrors and curved mirrors. Planar mirrors preserve perspective views whereas curved ones only do so in very specific configurations. We will discuss planar mirrors first.

2.1 Planar Mirrors

2.1.1 Unfolding - A Convenient Way for Interpreting Image Formation in Planar Mirror Systems Our discussion is based on the *ray unfolding* procedure which we will introduce and apply to different mirror systems. Ray unfolding has its origins in the optical literature on prism systems where the resulting plots are known as *tunnel diagrams* [1]. In this technique, every mirror interaction is applied to the world instead of the ray. The result is a straight ray that passes through a sequence of virtual copies of the world that is equivalent to the bouncing ray in the real world. This way, complex ray interactions can be visualized in an intuitive manner and a change of coordinate systems can easily be tracked.

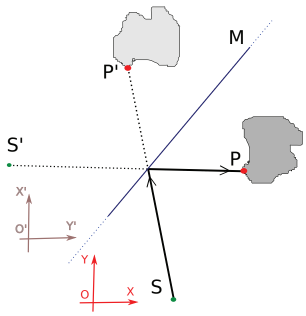

Fig. 2: Unfolding of a single reflection.

2.1.2 Single-Mirror, Single-Bounce Consider a single planar mirror and a camera observing an object via a single-bounce reflection, Fig. 2.

When a ray of light is hitting the mirror it is mirrored from the plane according to the law of reflection. Instead of mirroring the ray, we can consider that the world is being reflected, creating a virtual world, or as we well call it, a virtual *chamber*. In this case, the ray appears to continue straight into the virtual mirror world. The mirror copy of the scene is an isometric transformation of the real world. The world coordinate system is transformed to the mirrored one by reflecting it in the mirror plane. Left-handed mirror system transform into right-handed ones and vice versa. The procedure of ray straightening just described is called *unfolding*. Because light paths are reversible, we can consider the ray straightening procedure from the point of view of a scene point or from the point of view of a camera or a projector. Consider a ray from camera S observing a scene point P through the reflection from the planar mirror M. Then from the point of view of the camera, we observe the virtual point P' which is the mirror copy of the real point P. But from the point of view of the point P we are observing the virtual camera S' which is the reflection of the real camera S.

2.1.3 Multi-Mirror, Single-Bounce per Mirror If there are several planar mirrors that are arranged around a camera, as for example in Fig. 3 (left), for rays hitting different mirrors the ray straightening process will introduce a different virtual world (or a different virtual camera if we consider the point of view from the scene). A second possibility is to arrange the planar mirrors such, that there is a sequencial ray bouncing from mirror to mirror as shown in Fig. 3 (right). In this case the unfolding procedure is applied recursively. Thus, if an even number of reflections is involved, the

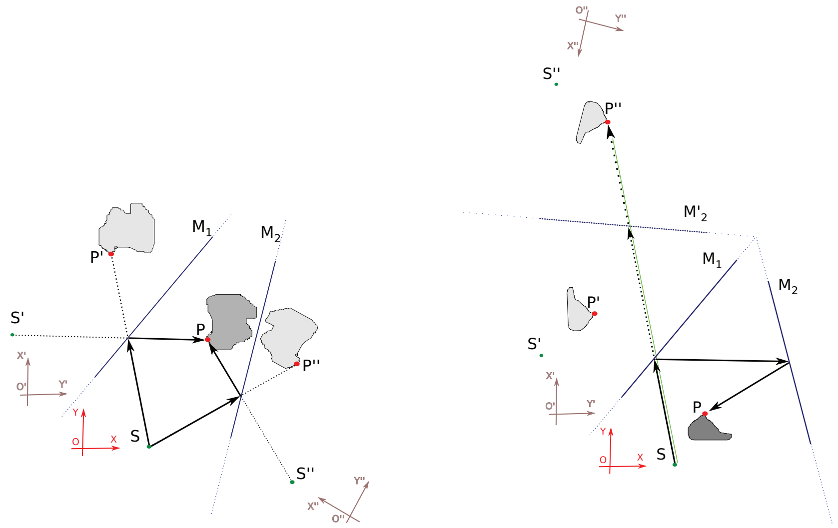

Fig. 3: Two planar mirrors: unfolding for two different rays (left) and unfolding for sequential reflection (right).

resulting virtual world (virtual camera) coordinate system will not change its handness while it changes handedness if the reflection level is odd.

As long as the reflection sequence includes every mirror only once, the recursive unfolding procedure can be applied without ambiguity.

2.1.4 Multi-Mirror, Multi-Bounce However, multiple bounces in systems with several planar mirrors could be such, that the same mirrors are participating in a reflection sequence multiple times. In a theoretical setting, this number could well be infinite.

The simplest such system is an angle constructed from two planar mirrors as in Fig. 4. There are several cases to consider that are instructive for the further discussion. If the angle $\angle ABC$ between the mirrors is $\frac{\pi}{k}$, where $k \in \mathbb{N}$, then the unfolding of all possible rays will introduce a partitioning of the space into continuous regions such that the space is divided into $2k$ different parts. These are the inner part of the original angle (base chamber) and the copies associated with different reflection levels (virtual chambers). The partitioning is, in this case, independent of the origin of the ray. A useful result that can immediately be verified in the unfolded representation is that no ray can have a sequence of more than k bounces.

Note, that the ray can hit either of the mirrors first. Therefore, different reflection sequences will occur to the left or to the right of the half-line BE. This half-line cuts some of the virtual chambers and its position depends on the location of the projective center S. In general, the unfolded space will be discontinuous across this half-line. For this reason, we call such a lines *discontinuity lines* [2].

In the example in Fig. 4, the discontinuity line BE is irrelevant because the chambers that are crossed by the discontinuity line BE overlap perfectly and, moreover, their transformed coordinate systems are the same. However, this is only the case if

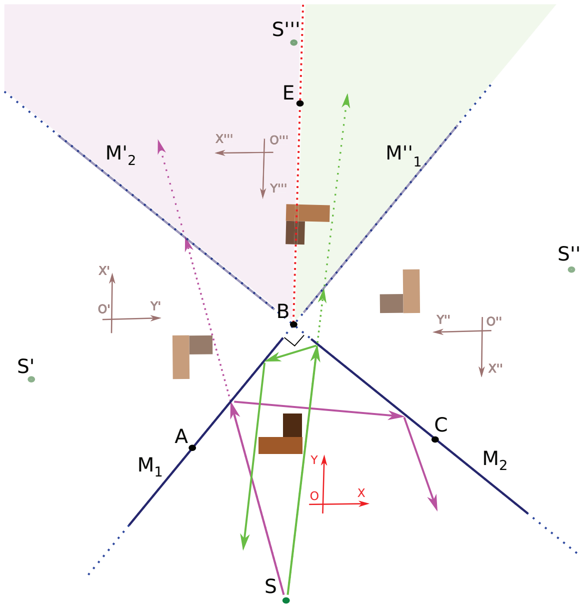

Fig. 4: Ray bouncing inside an angle with matching coordinate systems.

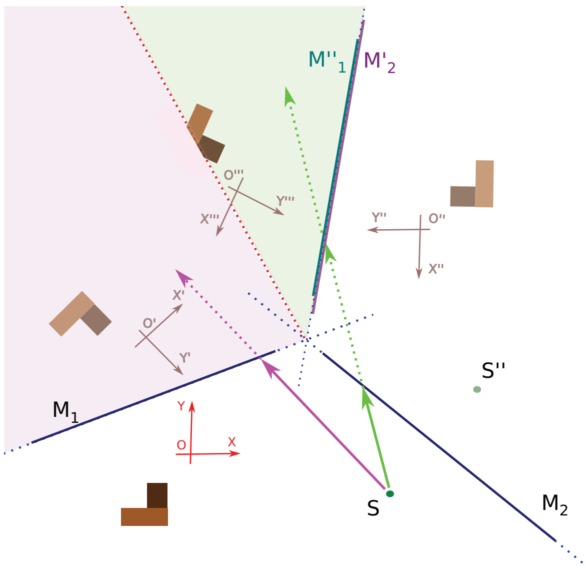

Fig. 5: Two different rays bouncing inside an angle where the chambers are matching but the coordinate systems do not.

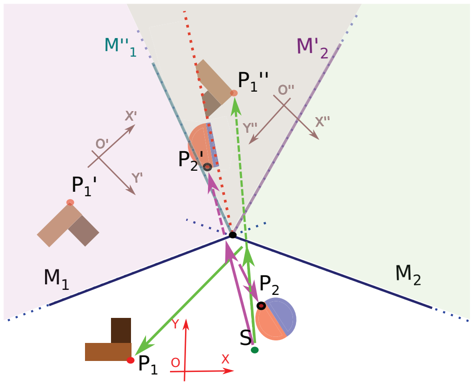

Fig. 6: Two different rays bouncing inside an angle where neither the coordinate systems nor the chambers match.

the angle between mirrors is exactly $\frac{\pi}{k}$. As another example, if the angle between the mirrors is $\frac{2\pi}{2k+1}$, then all the chambers still overlap perfectly, but the coordinate systems are not the same, see picture Fig. 5. Even worse, if the angle between the mirrors is not an integer fraction of 2π, then the virtual chambers do not match properly and their coordinate systems do not align, Fig. 6.

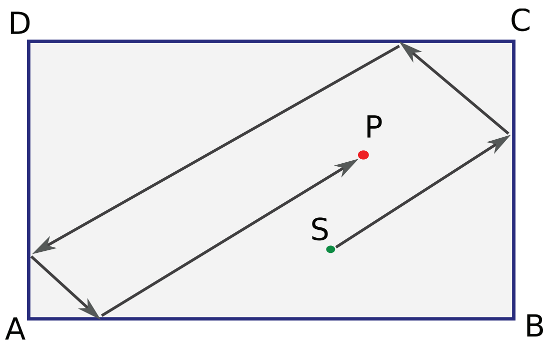

Fig. 7: Ray bouncing inside the rectangle ABCD. Light propagates from point S up to point P.

A simple example involving several mirrors is a bouncing ray inside a rectangular room, see Fig. 7. This type of geometry is most often considered in multi-bounce time-of-flight image, Sect. 6. If we repeatedly unfold the ray while it is propagating in space, we obtain the result seen in Fig. 8. In every virtual rectangle (virtual chamber) we have a virtual world that is specific to the sequence of reflections. If we consider all possible ray directions from any possible inner point of the original rectangle, we obtain a partitioning of the space into virtual rectangles. Since the rectangle is a regular structure,

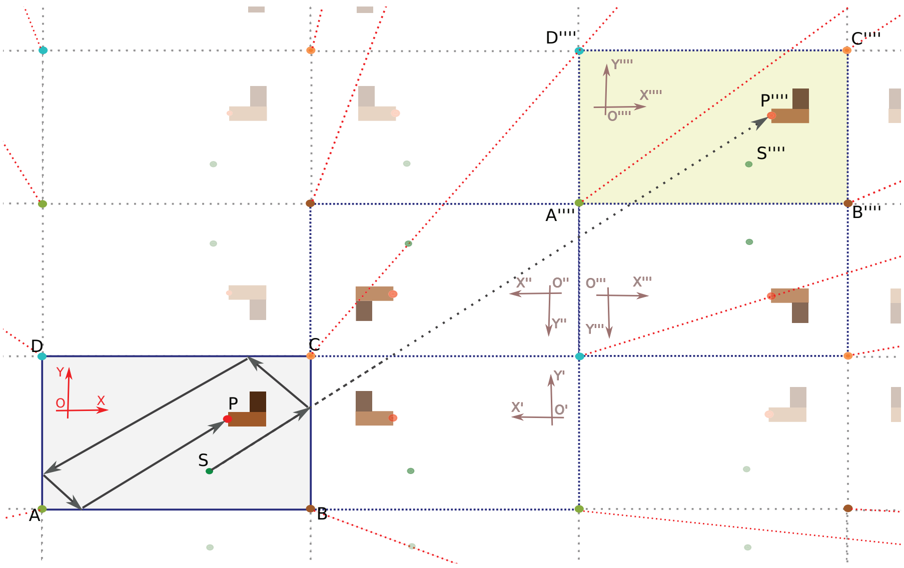

Fig. 8: Unfolding of the ray from Fig. 7 bouncing inside the rectangle.

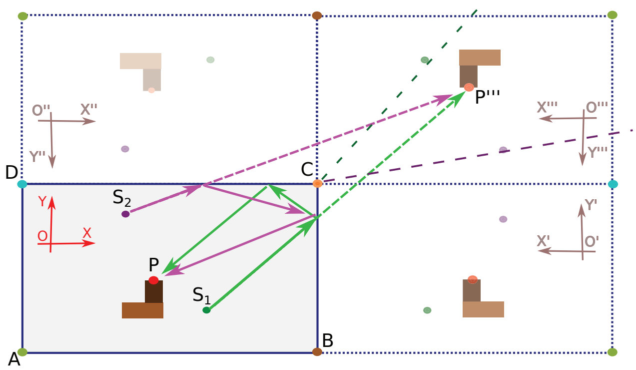

Fig. 9: Ray bouncing from two different camera locations, S_1, S_2 to the same object point P.

unfolding via different reflection sequences yields the same virtual worlds (perfectly overlapping chambers and equal coordinate system), independent of the sequence of reflections we travel along the ray to reach the virtual rectangle from the real one (see Fig. 9).

Unfortunately, only few types of polygons produce this similarly perfect space partitioning schemes. In these cases, the partitioning is independent of the initial ray position. The polygons (or polyhedra in the $3D$ case) having this property are known as Coxeter polygons (polyhedra). A polygon is a Coxeter polygon iff all its angles are in the form of $\frac{\pi}{k}$, $k \in \mathbb{N}$. There are only 4 such polygons: rectangles, equilateral triangles, the isosceles right triangles, and right triangles with angles $\frac{\pi}{3}$ and $\frac{\pi}{6}$.

For polyhedra in $3D$, the condition to be a Coxeter polyhedron is that all the dihedral angles are of the form $\frac{\pi}{k}$, $k \in \mathbb{N}$. There are only 7 types of Coxeter polyhedra [3].

All other types of polygons and polyhedra generate a more complicated space partitioning that depends on the ray origin [2].

2.2 Curved Mirrors

Curved mirrors are different from planar ones in the sense that they usually do not yield perspective views (except in special configurations) but rather transform the world according to their surface curvature. One can consider the curved mirror as a surface, that, at each point, has a corresponding planar mirror that is tangent to the surface. In order to use such mirrors in practice, their geometry and pose with respect to a recording camera or a projector has to be known very accurately. It is a difficult problem to estimate general mirror shapes precisely, Sect. 5. Therefore, in practice, only a limited number of mirror shapes are considered. The classes of mirrors utilized in practical settings, Sects. 3 and 4, are restricted to conic sections and to axially symmetric mirrors. In the following we classify these simple types of curved mirrors into the following groups

- General axial symmetric mirrors,
- Circular cone mirrors,
- Spherical mirrors,
- Elliptic mirrors,
- Parabolic mirrors,
- Hyperbolic mirrors, and
- Cylindrical mirrors,

and discuss their properties that are useful in imaging applications.

2.2.1 Single-Mirror, Single-Bounce Since all of these mirror shapes are axially symmetric, we start with a discussion of the general properties of axially symmetric systems.

2.2.1.1 General axial symmetric curved mirrors For any axially symmetric surface, an intersection with a plane orthogonal to the axis of symmetry is a circle. Therefore, if a projective center is placed on the axis of symmetry, rays through the projective center with the same angle towards the symmetry axis intersect the surface in the same circle. Moreover, the surface normal of the mirror at the intersection point is in the same plane as its symmetry axis and the propagation direction of the intersecting ray.

The law of reflection implies that the reflected ray is contained in this same plane. Thus, the process of reflection can be described in terms of reflections from curved mirrors within this plane ($2D$). In addition, all planes containing the symmetry axis of the mirror yield the same $2D$ profile, or, in other words, the reduced description is independent of the initial ray direction. Ray propagation is rotationally invariant for the mirror's axis of symmetry.

Consider one of the rays with the origin S placed on the axis of symmetry l of an axially symmetric curved mirror and with propagation direction $\boldsymbol{d}$, Fig. 10 (left). After reflection from the mirror surface, the ray changes its direction to $\boldsymbol{d'}$. Let S' be the intersection of a line with its origin at the intersection point and slope in the new propagation direction $\boldsymbol{d'}$ with the symmetry axis of the mirror l (we exclude the case, when the direction $\boldsymbol{d}$ coincide with l). S' can be considered as a virtual origin of the reflected ray. Because the ray propagation is rotationally invariant with respect to the axis of rotation l, any ray leaving S at the same angle (w.r.t the symmetry axis) as $\boldsymbol{d}$ has the same virtual origin S after reflection. We call the point S' the virtual focus of the ray bundle with angle α. In general, different angles result in different virtual focii, except for some special cases discussed below. The situation is depicted in Fig. 10 (middle and right).

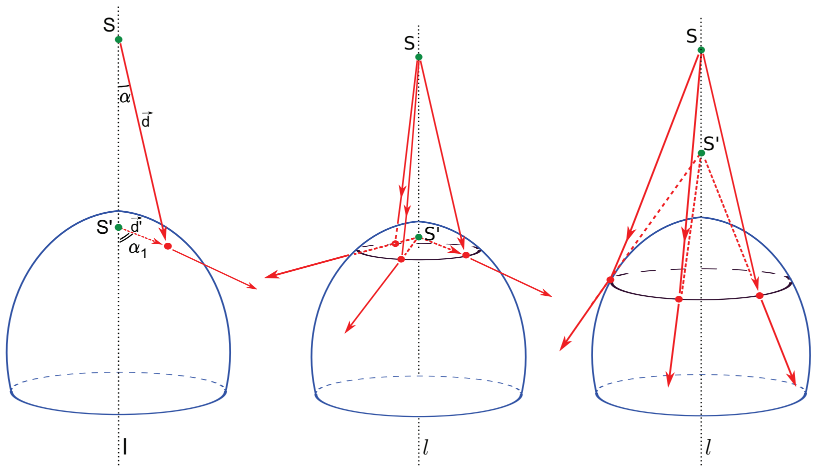

Fig. 10: Two different virtual foci together with a cones of rays propagation illustration

Another important property is related to convex curved axially symmetric mirrors (but it is also valid for an arbitrary convex mirror). If the surface of the mirror is convex, then the ray inclination angle at the virtual focus is larger, than the inclination angle at the real focus (see the angles α_1 and α on Fig. 10 (left)). In other words, convex mirrors widen the field of view.

This property is widely used in omnidirectional imaging devices, Sect. 3.2.1, and all subtypes of curved mirrors discussed below show this characteristic.

2.2.1.2 Circular cone mirrors Circular cone mirrors are axially symmetric mirrors with a linear cross-section, see Fig. 11. Thus, propagation of rays inside such a mirror can be translated to the propagation of rays in the $2D$ angle that was previously described and to which the unfolding procedure can be applied.

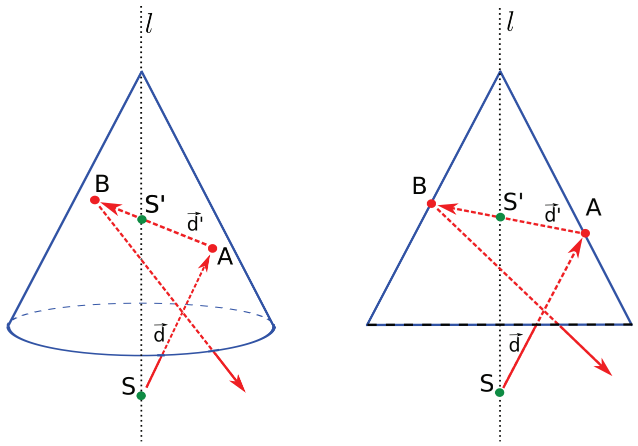

Fig. 11: Traversal of a ray inside a circular cone. 3D view (left) and corresponding plane projection (right).

2.2.1.3 Spherical mirrors The sphere is rotationally invariant with respect to its center. Therefore, the spherical mirror is axially symmetric with respect to any axis that intersects the center of the sphere. This is a very useful property as it solves the problem of adjusting the position of the projective center to match the symmetry axis. The second property of a spherical mirror is that all rays emanating from the center of the sphere are reflected back towards this point.

2.2.1.4 Elliptic mirrors The specific optical property of an ellipse is that light, exiting at one focus of the ellipse is reflecting such, that it passes through the other focus of the ellipse. Thus, if a perspective camera/projector is placed in one focus, it will observe/highlight the other focus for all light directions. Of course, in $3D$, the same properties apply to the ellipsoid, Fig. 12 (left).

2.2.1.5 Parabolic mirrors In a parabolic mirror, rays parallel to the main axis are reflected from the mirror surface to the mirror's focus and vice versa, Fig. 12 (middle).

2.2.1.6 Hyperbolic mirrors The useful property of a hyperbola is that rays emanating from one of its focii have a common virtual origin in the second focus when reflecting from the surface, Fig. 12 (right).

2.2.1.7 Cylindrical mirrors with curved cross sections The properties of a cylindrical mirror are dependent on its cross section (see Fig. 13). We will consider circular,

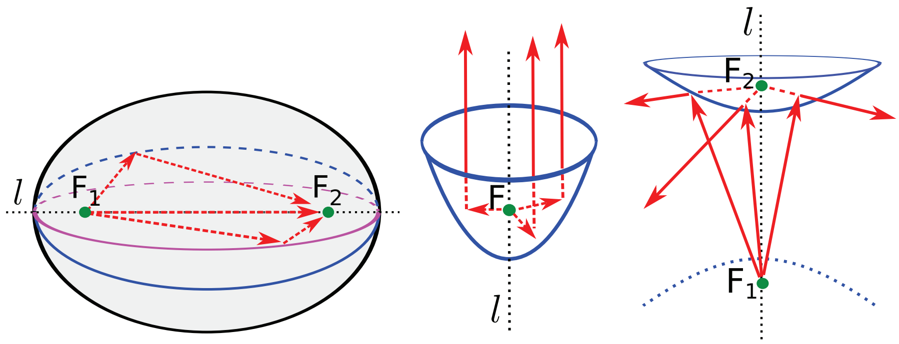

Fig. 12: Traversal of rays inside an ellipsoid (left), paraboloid (middle), and hyperboloid (right).

elliptic, parabolic and hyperbolic types of a cylinder cross sections. Each of these types inherits the properties of the corresponding $2D$ surface in such a way that the focii are elongated along the axis of the cylinder. Therefore, the propagation of the ray inside such a figure can be decomposed into two independent motions - one in the plane perpendicular to the cylinder axis, and the other along the cylinder axis. The first motion can be completely described by the propagation of the ray inside the $2D$ curve (circle, ellipse, parabola or hyperbola), while the second motion is constant.

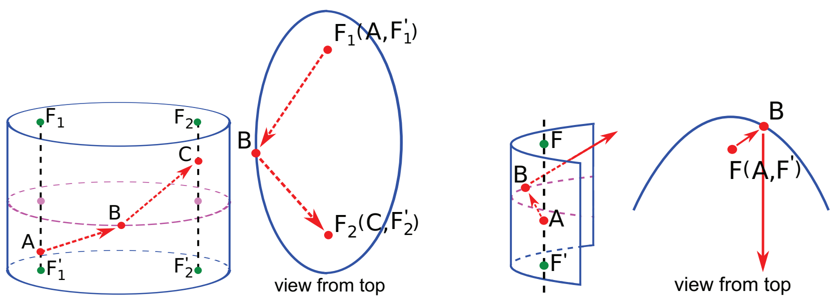

Fig. 13: Reflection of rays in cylindrical mirrors of elliptical cross section (left) and parabolic cross section (right).

3 Passive Imaging Systems

In this section we describe passive imaging devices that utilize mirrors in their design. The applications are mostly in stereo, multi-view and panoramic imaging. The advantages of employing mirrors in a system are usually

- a reduction in system cost by utilizing less sensor hardware,
- a simplification of synchronization by compressing several views onto a single sensor, and
- homogeneous radiometric and colorimetric properties of the sensor hardware.

In utilizing these advantages, sensor resolution is usually traded off for an expanded view point coverage of a scene.

3.1 Planar Mirrors

Planar mirrors are the simplest devices. As discussed in Sect. 2, single planar mirrors, and systems consisting of them have the advantage of preserving perspective projection properties at least in a subset of the pixels in an image.

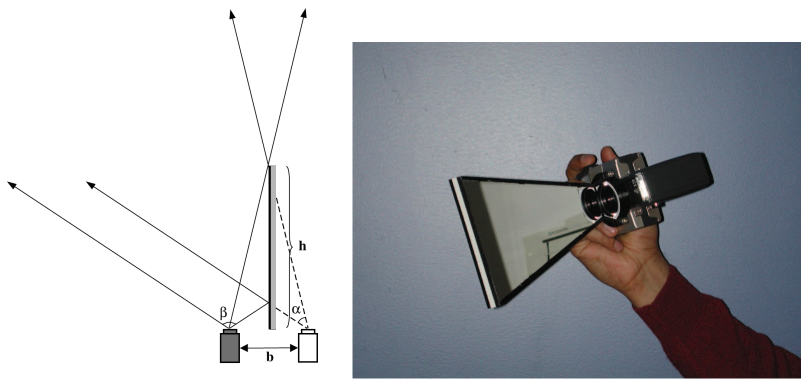

Fig. 14: Single mirror rectified catadioptric stereo camera [4]. (left) Image formation draft. (right) Camera prototype (http://www.cs.columbia.edu/CAVE/projects/cad_stereo/).

3.1.1 Single-Mirror, Single-Bounce Single planar mirror systems are necessarily single-bounce. They can thus be used to generate two viewpoints in a single image. This feature is often used to produce inexpensive stereo viewers in a dual screen setup [5] and many hobbyists make use of this capability http://klub.stereofotograf.eu/dual_monitor.php.

Similarly, a stereo camera can be built with a single mirror [6] and commercial modifications of standard cameras are being offered http://hineslab.com/old/Mirror_Stereo.html. Depending on the mirror orientation with respect to the camera optical axes, the resulting epipolar geometry can be more or less suitable for stereo matching. Gluckman and Nayar [4, 7] describe the conditions for epipolar lines to be parallel and along horizontal scan lines, a case that is particularly easy to handle in matching algorithms, see also Fig. 14.

In an early work, Mitsumoto et al. [8] describe object triangulation and geometric constraints for 3D reconstruction in case of a single plane mirror symmetry. They also time-sequentially move the mirror to different positions and merge the reconstructions to obtain a larger coverage of the object.

Moving planar mirrors are also used to inexpensively generate many viewpoints, e.g. for light field imaging [9] or 3D reconstruction [10, 11].

Beamsplitters are often employed to distribute a single view of a scene onto several imaging sensors. These devices can be considered as a special case of a single mirroring operation for one of the sensors, whereas the beamsplitter appears transparent to the other.

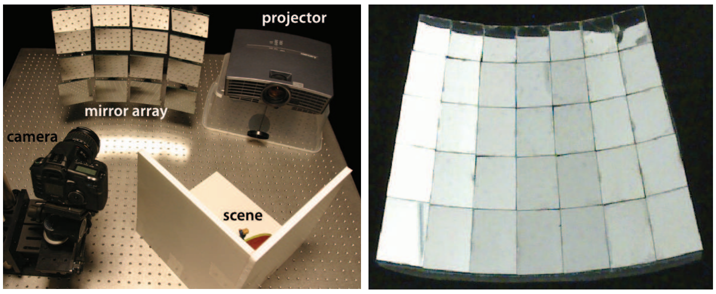

Fig. 15: A mirror array used for light field imaging [12] (left). A fabricated mirror array with an optimized facet distribution [13] (right).

3.1.2 Multi-Mirror, Single-Bounce per mirror

An increase in complexity and achievable imaging geometry is obtained when introducing several planar mirrors [4, 7]. Restrictions that guarantee a single bounce per mirror are a) that inter-reflections between mirrors are avoided, or b) that all camera rays only encounter mirroring sequences where each of the mirrors participates at most once.

3.1.2.1 No Inter-Reflections These arrangements are often employed for light field imaging with a single sensor [14, 12, 15, 13], see also Fig. 15. Since light field views differ only slightly from one another, mirror arrangements like the ones shown in the Figure can be suitably employed without too strong requirements on the positioning of the mirrors to avoid inter-reflections. Since views are usually supposed to cover a common viewing area, the carrier surface is chosen in a concave manner. If manufactured on a very small scale, faceted mirrors can be used to mimic bidirectional reflection distribution functions (BRDFs) with pre-defined properties [16].

Another way to avoid inter-reflections is to position planar mirrors on a convex surface [17, 18] and is realized using pyramidal or truncated pyramid structures. This measure yields out-ward facing views for panoramic imaging [18], or a means of performing aperture splitting of a single image onto several sensors [17], an application that is heavily used in computational photography applications.

In optics, in the area of multi-spectral imaging, especially manufactured mirrors, so called “image slicers” are being used to differently deflect the scan-lines of an image such that vertical sensor space is freed up for sensing spectrally expanded versions of the scan-lines that are obtained by passing them through a diffraction grating [19, 20, 21].

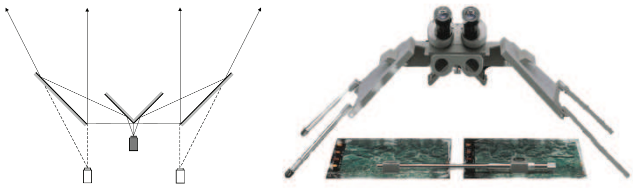

Fig. 16: Design for a four mirror stereo camera or viewing device [4] (left). The Sokkia MS27 commercial stereo viewer for aerial imagery (right).

3.1.2.2 Inter-Reflections with a Single Reflection per Mirror Several mirrors can also be arranged in a sequential sequence which yields a higher flexibility in generating virtual views and purely optical means of image manipulation. The most common commercial applications are probably erecting prisms in SLR view finders and other prism-based optical designs that are intended to flip or displace an image without distorting it otherwise [1].

However, several planar mirrors are also used to obtain a higher degree of flexibility in the design of stereo imaging systems [6, 4, 7] or in the production of stereo viewing equipment as e.g. produced by Sokkia, see also Fig. 16.

In computational photography settings, beamsplitter trees are often employed to deliver a single physical image to different sensor units. The optical path towards each of those sensor units can be modified such that optically differently filtered images are recorded. For an overview of this area the interested reader is referred to [22, 23, 24].

It should be mentioned that all applications discussed so far can be handled with the basic unfolding technique, Sect. 2.1.1.

Fig. 17: The five-view case employing inter-reflections up to second order [25] – self-occlusion is clearly visible (left). In the case of many inter-reflections a pixel labeling procedure is necessary [2] that can resolve the view assignment to pixels – up to eight reflection levels have been employed (right).

3.1.3 Multi-Mirror, Multi-Bounce Multi-bounce planar mirror systems are considerably more difficult to comprehend and to make use of. Early work in mirror-based single-image 3D reconstruction focused on setups consisting of two mirrors arranged such that their normals are in a common plane and that the angle between them is equal to $2\pi/N$. This has been a popular choice for three-dimensional imaging with a single camera with $N = 5$ views [26, 27, 25, 28]. It should be mentioned that this geometry results in a non-Coxeter structure and therefore the camera position has to be suitably chosen to hide discontinuous views, see Sect. 2.1.4. The multiple view geometry of this setting has been explored in [29].

A common problem with this arrangement, and in fact with any multi-bounce system, is that the object position has to be chosen very carefully. The problem that occurs in the multi-bounce case is that an object might occlude its virtual counter parts, an effect that is easily observed when viewing one-self in a set of opened bathroom mirrors. A solution to this problem has been presented recently [2] and consists in a pixel labeling procedure that determines for every pixel of an image with multiple inter-reflections which virtual view it belongs to, see also Fig. 17. This assignment can be computed from a single image and for arbitrary calibrated planar mirror geometries. Because of the kaleidoscopic nature of the resulting images, these systems are referred to as kaleidoscopic imaging systems.

3.2 Curved Mirrors

Imaging systems employing curved mirrors commonly aim at achieving a larger field-of-view than is possible with refractive optics at a reasonable price and with acceptable distortions. In particular, wide-angle refractive elements often suffer from strong aberrations.

3.2.1 Single-Mirror, Single-Bounce The most common use of single curved mirrors in conjunction with a camera device is omnidirectional or panoramic imaging. The applications of omnidirectional cameras are mainly in robotics for navigation purposes [30], omni-directional stereo if multiple images are available [31], teleconferencing [32] and panorama construction. Several companies such as Olympus and Canon have developed prototypes and other companies such as 0-360, GoPano, Neovision, RemoteReality, FullView Inc. and VersaCorp. are offering systems commercially. The technology has been employed in Microsoft's RoundTable.

Research efforts have been concentrated on determining adequate mirror shapes. While conic sections provide a single center of projection if the camera is placed in one of the focii for hyperbolic or ellipsoidal mirrors, see Sect. 2.2.1, a careful alignment of the camera and the mirror have to be performed. A more practical arrangement is the combination of an orthographic camera with a parabolic mirror [32]. This way, only the optical axis of the camera and the axis of the mirror have to align. The advantage of single center of projection systems is that proper projective views can be synthesized from the acquired imagery.

The design of non-center of projection systems has focused on achieving desirable properties of the observed projections. An inevitable feature of observations via curved mirrors are the distortions introduced by the curved surface. Some research has been performed on optimizing mirror shapes such as to achieve desirable projection properties such as a linear dependence between the incidence angle of world rays to radial image coordinates [33], the preservation of world space linearity in the images observed by the catadioptric system [34], the achievement of a pre-defined projection pattern [35], the capture of non-distorting wide-angle views [36], or the minimization of image space errors [37]. A common approach is the specification of derivative properties of the mirror surface, followed by solving a differential equation [33, 34, 37] or minimizing an error functional [35]. The concepts have been extended towards systems of catadioptric cameras [38].

When several cameras are available, catadioptric stereo matching can be performed [31] and depth maps can be computed. The associated research questions are related to the imaging geometry and therefore the calibration of such systems, Sect. 5.

An important aspect to be taken into account when using curved mirrors for imaging applications is their inherent property to refocus light rays. Studies concentrating on the effects of mirror-induced defocus characterization can be found in [39, 40, 41].

3.2.2 Single-Mirror, Multi-Bounce Curved mirrors are typically designed such that only a single reflection occurs for each camera ray. This is achieved by employing convex mirror shapes. However, it is possible to use multi-bounce mirror systems advantageously. In particular, cylindrical mirrors with a specular interior can be seen as a kaleidoscopic imaging system that is continuous in one dimension, whereas offering discrete view points in the other. They have been used for omnidirectional texture acquisition and depth estimation [42]. Cross-sections of this type of mirror can be analyzed with the unfolding procedure, Sect. 2.1.1.

3.2.3 Multi-Mirror, Single-Bounce Systems utilizing multiple curved mirrors are designed such that inter-reflections are prevented, or the corresponding image regions are excluded from analysis. The most popular multi-mirror arrangement for curved mirrors consists of arrays of mirror spheres [43, 44] or spherical caps [45, 46], the reason being that the sphere is rotationally symmetric around its center, thus offering homogeneous viewing properties for a single perspective camera observing several of them. Applications include light field imaging and 3D reconstruction. An initial study of the latter utilized two spherical mirrors observed by a perspective camera and described the resulting epipolar geometry for stereo matching [47]. A linear array of mirroring spheres was used to calibrate the position of a point light source [48]. A study of two conic sections imaged by a single perspective camera and of the resulting epipolar geometry is found in [49].

Panoramic cameras can be made more compact if reflection of two or more mirrors is permitted. The camera is again arranged along the symmetry axis of the mirror system. In the case of conic sections, it was shown that multi-mirror systems of such shapes always have an equivalent single mirror interpretation [50]. In [51], a double mirror system of the shape previously discussed is designed such that a stereo pair is formed in a single panoramic 360° image.

Arrays of mirror spheres have primarily been employed for light field imaging and 3D reconstruction. In [43, 44] the incident light field of scene illumination is acquired with an array of mirror spheres. The spatially and directionally varying illumination is then used to relight synthetic scenes [43] or to compute depth maps and perform refocusing operations [44]. Arrays of spherical caps reduce the unusable area at the sphere boundaries that suffer from inter-reflections. They have been used for light field imaging and 3D reconstruction [52, 45, 46].

4 Active Imaging Systems

Active imaging system employ a light source in addition to an imaging device. Nowadays, these light sources are typically digital projectors which enable a per-pixel control of the illumination. The use of combinations of cameras and projectors enables applications such as corrected projection onto curved surfaces, virtual large scale projection displays, 3D structured light scanning, reflectance scanning and more. An overview of the area of camera-projector systems is given in [53].

The combination of light sources with mirrors introduces additional problems in a measurement setting. Emitted light can super-position in a scene [54, 55], defocus problems [39, 40, 41] are exaggerated since projectors typically employ large apertures for light efficiency. On the other hand, active light helps in coding a scene, as e.g. in structured light scanning, or enables the scanning of surface properties.

4.1 Planar Mirrors

Planar mirrors are most often used to multiply the number of physical projectors or to virtually position them in a physically impossible location.

4.1.1 Single-Mirror, Single-Bounce The most common use of a single planar mirroring device is the use of a beamsplitter to bring a projector and a camera into a coaxial arrangement [56, 41, 57, 58, 59, 60]. This configuration allows for illumination along the same rays that form the camera image and is often part of more complex active imaging systems.

In a different application, the use of a single planar mirror for range scanning inaccessible parts of an object has been reported [54]. To avoid the super-position of light, the operator has to manually ensure that the real and virtual laser lines are formed in distinct regions and that a distance heuristic can distinguish between the 3D points generated in the real space and in the virtual space, respectively.

4.1.2 Multi-Mirror, Single-Bounce In the active setting, systems of planar mirrors multiply a single projector into a set of virtual projectors, in effect realizing a large aperture projection system. These virtual large apertures have been employed in synthetic aperture confocal imaging techniques [14, 15] where the superposition of light is a crucial part of the functioning of the device. Confocal imaging systems can slice a volumetric scene via very shallow depth-of-field imaging and illumination. The planar mirrors are arranged tangent to a concave base shape [14] which is ellipsoidal in the case of [15]. The mirror array is simultaneously used as a light field imaging unit, Sect. 3.1.2. The geometrical layout and interpretation are as discussed in Sect. 2.1.3.

Sequential folding of projection cones is often employed in rear-projection screens to reduce the size of the room that is required behind the screen. Typically, large-scale front-surface mirrors are employed for this purpose, `http://www.screen-tech.eu`.

4.1.3 Multi-Mirror, Multi-Bounce As mentioned in Sect. 3.1.2, the main complication in utilizing multiple ray bounces in a mirror system is that self-occlusion between the object and its virtual counter-parts has to be avoided. The simplest solution to this problem is the imaging of flat objects [61, 62]. In [61], a kaleidoscopic mirror system was introduced that was capable of scanning the bidirectional texture function (BTF), also known as spatially varying BRDF, of a surface without moving the acquisition apparatus or the sample. In this case it is possible to observe a surface light field with a single picture and the sample can be illuminated from different directions by using a digital projector that is only highlighting specific chambers. A sampling analysis of this type of system can be found in [62].

Kaleidoscopic reflectance scanning has been extended to take extended depth objects into account [55]. The solution is similar to the pixel labeling procedure [2], Fig. 17 (right), this time applied to the projector coordinate system. If only pixels that have a unique label are illuminated simultaneously, the virtual illumination is guaranteed to come from a single direction without causing illumination overlap in the scene. The authors combined reflectance scanning with omnidirectional laser-range scanning.

The superposition of light can also be arranged such that a projected pattern perfectly super-positions onto itself. This approach requires orthogonal illumination with a direction that is contained in the plane spanned by the mirror normals. The two-mirror/five-virtual view system mentioned in Sect. 3.1.2 has been used for this purpose [25, 28].

4.2 Curved Mirrors

Curved mirrors are not widely used for projection purposes, most likely due to defocusing of the projected image. For this reason, apparently only small aperture "pocket projectors" use this technology.

4.2.1 Single-Mirror, Single-Bounce An example for a curved mirror in a commercial projector is the RICOH PJ WX4130. It is used to achieve a very short focusing distance and a large field-of-view, `http://www.ricoh.com/about/company/technology/tech/040.html`. In research, curved mirrors have been investigated for achieving large fields of view [63] and optical undistortion when projecting onto tilted or curved walls [38].

In the area of reflection measurement, a combination of orthographic camera, collimated light source and a parabolic mirror section allows for convenient scanning of the BRDF of flat samples [64, 65]. When the mirror is moved, the surface can be scanned and a spatially varying BRDF is measured. The directional scanning is performed anglewise by using different projector pixels. A similar system can be built from elliptical mirrors and a perspective camera, putting the material sample in one focus and the projector-camera system in the other [66]. The latter reference also employs per-angle scanning, however, the system has been extended to multiplexed illumination [67].

4.2.2 Single-Mirror, Multi-Bounce Single curved mirror active multi-bounce systems have not been explored to our knowledge.

4.2.3 Multi-Mirror, Single-Bounce A combination of two curved mirrors and a coaxial camera/projector configuration has been used to perform BRDF measurements directly in some basis [58, 59]. The basis is projected into the system and the observations consist of the integral between the incident illumination and the BRDF. This integral corresponds to a scalar product of the BRDF and the basis illumination and can therefore be used to measure a basis expansion of the BRDF signal, permitting to sample less coefficients if the reflectance is low frequency or the basis of the material is known.

4.2.4 Multi-Mirror, Multi-Bounce Multiple curved mirror active multi-bounce systems have not been explored to our knowledge.

5 Mirror Calibration and Geometry Reconstruction of Specular Surfaces

In order to successfully use mirror systems, they have to be calibrated. Usually this involves the estimation of the mirror position and orientation, potentially its shape, and its radiometric properties [2, 55].

5.1 Planar Mirrors

Planar mirrors are relatively simple to calibrate since they do not introduce additional distortions into the image. Instead, the image taken by a perspective camera shows different perspective sub-views in parts of the acquired image. It is therefore only necessary to determine the image regions that correspond to a particular view, a task that is often performed manually. Within these viewing regions, standard perspective camera calibration techniques can be employed [68]. In the case of single bounce observation, this calibration is usually sufficient.

5.1.1 Single Mirror, Single-Bounce In case of a moving mirror, it is usually necessary to estimate the mirror pose with respect to the recording camera, since an offline calibration step cannot easily be employed. For this purpose, self-identifying markers that are attached to the mirror can be used [9]. Moving platforms are also often employed in the case of robotic applications. The case of a two-planar mirror setup with a moving camera mounted on a robotic platform has been analyzed in [69]. The authors derive a calibration procedure for computing the pose of the camera with respect to the mirrors as well as the mirrors' relative position and orientation.

5.1.2 Multiple Mirrors, Multi-Bounce In the case of multi-bounce observation, the mirror poses as well as the single real camera pose need to be estimated very accurately since the calibration error increases exponentially with the level of reflection. For this reason, special calibration procedures are necessary. In [70] a fixed (and known) mirror geometry is assumed and an algorithm for pose recovery of the real camera that is based on scene point correspondences (without knowing their reflection level) is derived.

For kaleidoscopic imaging systems, different procedures can be used. For systems that are only imaging flat samples, it is sufficient to determine the homographies mapping the acquired views to their rectified versions [61]. The geometry of the mirror system and the camera need not to be known. For kaleidoscopic systems that image extended objects, it is necessary to estimate the mirrors' positions and orientations with respect to the real camera. In [2, 55] this is done by placing a checkerboard pattern at different heights into the system and manually identifying the direct view of the camera and the first order reflections. From this data, initial mirror parameters can be computed that are then used to predict higher order reflections. Incorporating this new information yields improved results. A global bundle adjustment step finishes the procedure.

The manual identification of reflection levels in a multi-bounce image is tedious and error prone. In [71], an automatic procedure is proposed that can recover the number of mirrors and their parameters without user intervention. Unfortunately, the method is restricted to $2\frac{1}{2}D$ settings.

5.2 Curved Mirrors

The calibration or geometry estimation of general curved mirrors or specular surfaces of general shape is a difficult subject. A recent review article covering the area can be found in [72]. When using curved mirrors in imaging systems, it is necessary to calibrate the shape parameters of the mirror as well as its pose with respect to the camera.

5.2.1 Single Mirror, Single-Bounce The single-bounce case for parametric mirrors is the most investigated class of algorithms for mirror calibration. A good overview of imaging, pose estimation, and multiple view geometry that includes specific sections on catadioptric systems can be found in [73].

A prerequisite for investigating calibration problems is an understanding of the imaging geometry of such systems.

5.2.1.1 Imaging Geometry An analysis and classification of distortions in multi-perspective images and a corresponding undistortion algorithm are discussed in [74]. The concept of general linear cameras has been introduced as a general piece-wise linear class of imaging models for multi-perspective images. Its application to reflection modeling and an overview of its applications are given in [75].

The class of conic section mirrors has received the widest attention. Viewpoint caustics of curved mirrors are intrinsically linked to their imaging properties and are discussed in [76]. The imaging geometry of central catadioptric systems, i.e. those featuring a single center of projection are discussed in [39, 40, 77, 78]. Central systems require the camera to be in a specific point with its optical axis aligned with the mirror axis (except for the parabolic mirror/orthographic camera case). If the camera's optical axis is aligned but the camera is not in the correct location, the system is called non-central (because it does not have a single center of projection, see Sect. 2.2.1). For this case analytic forward projection models have been derived [79] that result in higher order polynomial formulations. The condition for the camera to be on axis has recently been lifted [80].

5.2.1.2 Shape Estimation Imaging geometries as described above can be utilized to derive shape recovery algorithms. These usually aim at recovering the parameters of a conic section known as the intrinsic parameters of the mirror system as well as the effective focal length of the combined mirror-camera system. Shape estimation for central catadioptric systems has been analyzed from the apparent distortion of scene lines in an image [77]. The calibration of a parabolic mirror/orthographic camera system is discussed in [81, 78]. The parameters of a conic section mirror can also be estimated from point correspondences between different frames of a moving camera [76].

Images of warped scene lines can also be used to compute the shape of more general shapes. In particular, near-flat specular surfaces can be recovered by locally fitting the general linear camera model [82, 83]. More generally, specular surfaces can be scanned by establishing scene plane-to-image plane correspondences [84, 85]. A naïve application of the principle results in a 1D ambiguity between the depth and the normal of the surface at one position. This ambiguity has recently been resolved [86].

5.2.1.3 Pose Estimation An associated problem in calibration is the pose estimation problem: Given a known camera/mirror configuration that is moved to different positions, what is the relative pose of the two views ? An early analysis of the egomotion problem with a central catadioptric is discussed in [30]. Here, optical flow in the recorded images is used to compute the trajectory of the camera. A similar problem based on sparse feature tracking was proposed in [87]. The epipolar geometry of central catadioptric cameras was investigated in [88, 89]. It allows for pose estimation and simplified stereo matching.

5.2.2 Multiple Mirrors, Single-Bounce Curved mirror arrays have mainly been used in the form of arrays of spheres, Sect. 3.2.3. Calibration methods use images of distorted checkerboards to infer the position and radii of the spheres with respect to the recording camera. A method for calibrating an array of spherical mirror caps positioned on a common ground plane is described in [45]. Recently, a method for calibrating several mirror spheres in general position has been developed [90].

5.2.3 Multiple Mirrors, Multi-Bounce The most general result so far on multiple specular surface interactions is that a maximum of two specular interactions of a ray with a general surface can be recovered, regardless of the number of measurements [91]. The article derives a theory of local specular surface interactions and derives tractable triangulation problems on a local per-ray basis. As input, the authors consider an arbitrary number of correspondences between several image planes and several world planes intersected by the ray in question.

6 Connection to Time-of-Flight Imaging and the Multi-Bounce Problem

The time-of-flight problem, at first hand, appears to be disconnected from the problem settings considered so far and in fact, the literature is largely orthogonal. In time-of-flight imaging, a pulse is emitted at one spatial location and the time difference until the signal returns is measured by the sensor. The classical time-of-flight technique is RADAR, where radio waves are used as probes. SONAR uses sound waves and LIDAR is using light pulses, usually infrared, to determine the distance of objects. In pulse-based time-of-flight imaging, most commonly, a single reflection of the emitted pulse from the environment is assumed. This situation is equivalent to a single-mirroring operation. In practice, multiple echoes, or multi-bounce signals, can corrupt the detection. Most often, these echoes are considered to be undesirable noise and filtering procedures are developed to identify first time of arrivals, see [92, 93] and the references therein.

Multi-bounce analysis in this area is investigating the forward modeling of reverberation and recovery of a room geometry from impulse responses of a room. The forward modeling frequently employs unfolding procedures, Sect. 2.1.1, for Coxeter geometries [94], or for arbitrary polyhedral models [95].

The recovery of room geometries from multi-bounce data often considers the special case of a rectangular Coxeter geometry [96] also known as the *shoebox* model, which allows for the interpretation as a perfectly sub-divided space. Only recently methods for general convex geometries have started to appear ([97] and the references therein). These methods usually assume the first-bounce, other reflection levels, or the number of walls of the room to be known. Two recent methods that do not use these assumptions are [98, 71].

The computer vision literature on time-of-flight is presented elsewhere in this book. We would still like to point out recent developments that enable the recording of the temporal profile of light for every pixel [99]. While the initial *transient imaging* work used a very expensive femto-second laser setup, recently the use of a standard time-of-flight imager for the measurement of transient images has been proposed [100]. The information acquired with these devices can be used to reconstruct geometry from indirectly observed bounces, i.e. the geometry of hidden objects [101].

7 Conclusions

We have reviewed and classified the literature regarding the use of mirror systems in computer graphics and computer vision, and established some connections to the area of time-of-flight imaging. Some possible configurations of mirror systems have been identified as yet of unexplored, in particular, the use of multiple bounce curved mirrors in active imaging systems.

Another outcome of this review is that the design and optimization of mirror layouts is an area of study that has been only partially addressed so far. In particular, the global optimization of mirror system properties appears to be a promising, if challenging, research direction.

The analysis of multi-bounce reflection systems is more advanced in the computer graphics and computer vision literature than in time-of-flight imaging, where multi-bounce signals are predominantly considered as noise rather then a source of information.

Finally, we have discussed the, as of yet, little used tool of ray unfolding. It enables a simplified understanding of complex specular interactions and, in our opinion, can serve a useful role in exploring the area. It would be desirable to extend it such that more general classes of mirror shapes, especially non-central curved ones can be handled with similar ease as the planar multi-bounce case.

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